

# Appendix A: Literacy in History/Social Studies, Science, and Technical Subjects 6–12: College and Career Readiness Anchor Standards for Reading and Writing

## College and Career Readiness Anchor Standards for Reading

The grades 6–12 standards on the following pages define what students should understand and be able to do by the end of each grade span. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

### Key Ideas and Details

1. Read closely to determine what the text says explicitly and to make logical inferences from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
2. Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.
3. Analyze how and why individuals, events, or ideas develop and interact over the course of a text.

### Craft and Structure

4. Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.
5. Analyze the structure of texts, including how specific sentences, paragraphs, and larger portions of the text (e.g., a section, chapter, scene, or stanza) relate to each other and the whole.
6. Assess how point of view or purpose shapes the content and style of a text.

### Integration of Knowledge and Ideas

7. Integrate and evaluate content presented in diverse formats and media, including visually and quantitatively, as well as in words.\*
8. Delineate and evaluate the argument and specific claims in a text, including the validity of the reasoning as well as the relevance and sufficiency of the evidence.
9. Analyze how two or more texts address similar themes or topics in order to build knowledge or to compare the approaches the authors take.

### Range of Reading and Level of Text Complexity

10. Read and comprehend complex literary and informational texts independently and proficiently.

\*Please see “Research to Build and Present Knowledge” in Writing for additional standards relevant to gathering, assessing, and applying information from print and digital sources.

## Note on range and content of student reading

*Reading is critical to building knowledge in history/social studies as well as in science and technical subjects. College and career ready reading in these fields requires an appreciation of the norms and conventions of each discipline, such as the kinds of evidence used in history and science; an understanding of domain-specific words and phrases; an attention to precise details; and the capacity to evaluate intricate arguments, synthesize complex information, and follow detailed descriptions of events and concepts. In history/social studies, for example, students need to be able to analyze, evaluate, and differentiate primary and secondary sources. When reading scientific and technical texts, students need to be able to gain knowledge from challenging texts that often make extensive use of elaborate diagrams and data to convey information and illustrate concepts. Students must be able to read complex informational texts in these fields with independence and confidence because the vast majority of reading in college and workforce training programs will be sophisticated nonfiction. It is important to note that these Reading standards are meant to complement the specific content demands of the disciplines, not replace them.*

## Reading Standards for Literacy in History/Social Studies 6–12

RH

The standards below begin at grade 6; standards for K–5 reading in history/social studies, science, and technical subjects are integrated into the K–5 Reading standards. The CCR anchor standards and high school standards in literacy work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

| Grades 6–8 students:   | Grades 9–10 students:   | Grades 11–12 students:   |
|--|---|--|
| <b>Key Ideas and Details</b>   |   |  |
| 1. Cite specific textual evidence to support analysis of primary and secondary sources.  | 1. Cite specific textual evidence to support analysis of primary and secondary sources, attending to such features as the date and origin of the information.                       | 1. Cite specific textual evidence to support analysis of primary and secondary sources, connecting insights gained from specific details to an understanding of the text as a whole.   |
| 2. Determine the central ideas or information of a primary or secondary source; provide an accurate summary of the source distinct from prior knowledge or opinions.   | 2. Determine the central ideas or information of a primary or secondary source; provide an accurate summary of how key events or ideas develop over the course of the text.         | 2. Determine the central ideas or information of a primary or secondary source; provide an accurate summary that makes clear the relationships among the key details and ideas.  |
| 3. Identify key steps in a text's description of a process related to history/social studies (e.g., how a bill becomes law, how interest rates are raised or lowered). | 3. Analyze in detail a series of events described in a text; determine whether earlier events caused later ones or simply preceded them.  | 3. Evaluate various explanations for actions or events and determine which explanation best accords with textual evidence, acknowledging where the text leaves matters uncertain.  |
| <b>Craft and Structure</b>   |   |  |
| 4. Determine the meaning of words and phrases as they are used in a text, including vocabulary specific to domains related to history/social studies.                  | 4. Determine the meaning of words and phrases as they are used in a text, including vocabulary describing political, social, or economic aspects of history/social studies.         | 4. Determine the meaning of words and phrases as they are used in a text, including analyzing how an author uses and refines the meaning of a key term over the course of a text (e.g., how Madison defines <i>faction</i> in <i>Federalist</i> No. 10). |
| 5. Describe how a text presents information (e.g., sequentially, comparatively, causally).   | 5. Analyze how a text uses structure to emphasize key points or advance an explanation or analysis.   | 5. Analyze in detail how a complex primary source is structured, including how key sentences, paragraphs, and larger portions of the text contribute to the whole.   |
| 6. Identify aspects of a text that reveal an author's point of view or purpose (e.g., loaded language, inclusion or avoidance of particular facts).                    | 6. Compare the point of view of two or more authors for how they treat the same or similar topics, including which details they include and emphasize in their respective accounts. | 6. Evaluate authors' differing points of view on the same historical event or issue by assessing the authors' claims, reasoning, and evidence.   |
| <b>Integration of Knowledge and Ideas</b>  |   |  |
| 7. Integrate visual information (e.g., in charts, graphs, photographs, videos, or maps) with other information in print and digital texts.                             | 7. Integrate quantitative or technical analysis (e.g., charts, research data) with qualitative analysis in print or digital text.   | 7. Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, as well as in words) in order to address a question or solve a problem.  |
| 8. Distinguish among fact, opinion, and reasoned judgment in a text.   | 8. Assess the extent to which the reasoning and evidence in a text support the author's claims.   | 8. Evaluate an author's premises, claims, and evidence by corroborating or challenging them with other information.  |
| 9. Analyze the relationship between a primary and secondary source on the same topic.  | 9. Compare and contrast treatments of the same topic in several primary and secondary sources.  | 9. Integrate information from diverse sources, both primary and secondary, into a coherent understanding of an idea or event, noting discrepancies among sources.  |
| <b>Range of Reading and Level of Text Complexity</b>   |   |  |
| 10. By the end of grade 8, read and comprehend history/social studies texts in the grades 6–8 text complexity band independently and proficiently.                     | 10. By the end of grade 10, read and comprehend history/social studies texts in the grades 9–10 text complexity band independently and proficiently.                                | 10. By the end of grade 12, read and comprehend history/social studies texts in the grades 11–CCR text complexity band independently and proficiently.   |

## Reading Standards for Literacy in Science and Technical Subjects 6–12

RST

| Grades 6–8 students:  | Grades 9–10 students:   | Grades 11–12 students:   |
|---|---|--|
| <b>Key Ideas and Details</b>  |   |  |
| 1. Cite specific textual evidence to support analysis of science and technical texts.   | 1. Cite specific textual evidence to support analysis of science and technical texts, attending to the precise details of explanations or descriptions.   | 1. Cite specific textual evidence to support analysis of science and technical texts, attending to important distinctions the author makes and to any gaps or inconsistencies in the account.                  |
| 2. Determine the central ideas or conclusions of a text; provide an accurate summary of the text distinct from prior knowledge or opinions.   | 2. Determine the central ideas or conclusions of a text; trace the text's explanation or depiction of a complex process, phenomenon, or concept; provide an accurate summary of the text.                                   | 2. Determine the central ideas or conclusions of a text; summarize complex concepts, processes, or information presented in a text by paraphrasing them in simpler but still accurate terms.                   |
| 3. Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.  | 3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.                          | 3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.           |
| <b>Craft and Structure</b>  |   |  |
| 4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to <i>grades 6–8 texts and topics</i> . | 4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to <i>grades 9–10 texts and topics</i> .                | 4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to <i>grades 11–12 texts and topics</i> .  |
| 5. Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole and to an understanding of the topic.  | 5. Analyze the structure of the relationships among concepts in a text, including relationships among key terms (e.g., <i>force</i> , <i>friction</i> , <i>reaction force</i> , <i>energy</i> ).                            | 5. Analyze how the text structures information or ideas into categories or hierarchies, demonstrating understanding of the information or ideas.   |
| 6. Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text.   | 6. Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text, defining the question the author seeks to address.  | 6. Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text, identifying important issues that remain unresolved.                               |
| <b>Integration of Knowledge and Ideas</b>   |   |  |
| 7. Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).              | 7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words. | 7. Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.               |
| 8. Distinguish among facts, reasoned judgment based on research findings, and speculation in a text.  | 8. Assess the extent to which the reasoning and evidence in a text support the author's claim or a recommendation for solving a scientific or technical problem.  | 8. Evaluate the hypotheses, data, analysis, and conclusions in a science or technical text, verifying the data when possible and corroborating or challenging conclusions with other sources of information.   |
| 9. Compare and contrast the information gained from experiments, simulations, video, or multimedia sources with that gained from reading a text on the same topic.  | 9. Compare and contrast findings presented in a text to those from other sources (including their own experiments), noting when the findings support or contradict previous explanations or accounts.                       | 9. Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible. |
| <b>Range of Reading and Level of Text Complexity</b>  |   |  |
| 10. By the end of grade 8, read and comprehend science/technical texts in the grades 6–8 text complexity band independently and proficiently.   | 10. By the end of grade 10, read and comprehend science/technical texts in the grades 9–10 text complexity band independently and proficiently.   | 10. By the end of grade 12, read and comprehend science/technical texts in the grades 11–CCR text complexity band independently and proficiently.  |



## College and Career Readiness Anchor Standards for Writing

The grades 6–12 standards on the following pages define what students should understand and be able to do by the end of each grade span. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

### Text Types and Purposes\*

1. Write arguments to support claims in an analysis of substantive topics or texts using valid reasoning and relevant and sufficient evidence.
2. Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.
3. Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details and well-structured event sequences.

### Production and Distribution of Writing

4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach.
6. Use technology, including the Internet, to produce and publish writing and to interact and collaborate with others.

### Research to Build and Present Knowledge

7. Conduct short as well as more sustained research projects based on focused questions, demonstrating understanding of the subject under investigation.
8. Gather relevant information from multiple print and digital sources, assess the credibility and accuracy of each source, and integrate the information while avoiding plagiarism.
9. Draw evidence from literary or informational texts to support analysis, reflection, and research.

### Range of Writing

10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.

\*These broad types of writing include many subgenres. See Appendix A for definitions of key writing types.

### Note on range and content of student writing

*For students, writing is a key means of asserting and defending claims, showing what they know about a subject, and conveying what they have experienced, imagined, thought, and felt. To be college and career ready writers, students must take task, purpose, and audience into careful consideration, choosing words, information, structures, and formats deliberately. They need to be able to use technology strategically when creating, refining, and collaborating on writing. They have to become adept at gathering information, evaluating sources, and citing material accurately, reporting findings from their research and analysis of sources in a clear and cogent manner. They must have the flexibility, concentration, and fluency to produce high-quality first-draft text under a tight deadline and the capacity to revisit and make improvements to a piece of writing over multiple drafts when circumstances encourage or require it. To meet these goals, students must devote significant time and effort to writing, producing numerous pieces over short and long time frames throughout the year.*

## Writing Standards for Literacy in History/Social Studies, Science, and Technical Subjects 6–12

WHST

The standards below begin at grade 6; standards for K–5 writing in history/social studies, science, and technical subjects are integrated into the K–5 Writing standards. The CCR anchor standards and high school standards in literacy work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

| Grades 6–8 students:  | Grades 9–10 students:  | Grades 11–12 students:  |
|---|--|---|
| <b>Text Types and Purposes</b>  |  |   |
| <p>1. Write arguments focused on <i>discipline-specific content</i>.</p> <ul style="list-style-type: none"> <li>a. Introduce claim(s) about a topic or issue, acknowledge and distinguish the claim(s) from alternate or opposing claims, and organize the reasons and evidence logically.</li> <li>b. Support claim(s) with logical reasoning and relevant, accurate data and evidence that demonstrate an understanding of the topic or text, using credible sources.</li> <li>c. Use words, phrases, and clauses to create cohesion and clarify the relationships among claim(s), counterclaims, reasons, and evidence.</li> <li>d. Establish and maintain a formal style.</li> <li>e. Provide a concluding statement or section that follows from and supports the argument presented.</li> </ul> | <p>1. Write arguments focused on <i>discipline-specific content</i>.</p> <ul style="list-style-type: none"> <li>a. Introduce precise claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that establishes clear relationships among the claim(s), counterclaims, reasons, and evidence.</li> <li>b. Develop claim(s) and counterclaims fairly, supplying data and evidence for each while pointing out the strengths and limitations of both claim(s) and counterclaims in a discipline-appropriate form and in a manner that anticipates the audience's knowledge level and concerns.</li> <li>c. Use words, phrases, and clauses to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims.</li> <li>d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing.</li> <li>e. Provide a concluding statement or section that follows from or supports the argument presented.</li> </ul> | <p>1. Write arguments focused on <i>discipline-specific content</i>.</p> <ul style="list-style-type: none"> <li>a. Introduce precise, knowledgeable claim(s), establish the significance of the claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that logically sequences the claim(s), counterclaims, reasons, and evidence.</li> <li>b. Develop claim(s) and counterclaims fairly and thoroughly, supplying the most relevant data and evidence for each while pointing out the strengths and limitations of both claim(s) and counterclaims in a discipline-appropriate form that anticipates the audience's knowledge level, concerns, values, and possible biases.</li> <li>c. Use words, phrases, and clauses as well as varied syntax to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims.</li> <li>d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing.</li> <li>e. Provide a concluding statement or section that follows from or supports the argument presented.</li> </ul> |

## Writing Standards for Literacy in History/Social Studies, Science, and Technical Subjects 6–12

WHST

| Grades 6–8 students:   | Grades 9–10 students:   | Grades 11–12 students:   |
|--|---|--|
| Text Types and Purposes (continued)  |   |  |
| <p>2. Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes.</p> <ul style="list-style-type: none"> <li>a. Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories as appropriate to achieving purpose; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension.</li> <li>b. Develop the topic with relevant, well-chosen facts, definitions, concrete details, quotations, or other information and examples.</li> <li>c. Use appropriate and varied transitions to create cohesion and clarify the relationships among ideas and concepts.</li> <li>d. Use precise language and domain-specific vocabulary to inform about or explain the topic.</li> <li>e. Establish and maintain a formal style and objective tone.</li> <li>f. Provide a concluding statement or section that follows from and supports the information or explanation presented.</li> </ul> | <p>2. Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes.</p> <ul style="list-style-type: none"> <li>a. Introduce a topic and organize ideas, concepts, and information to make important connections and distinctions; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension.</li> <li>b. Develop the topic with well-chosen, relevant, and sufficient facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic.</li> <li>c. Use varied transitions and sentence structures to link the major sections of the text, create cohesion, and clarify the relationships among ideas and concepts.</li> <li>d. Use precise language and domain-specific vocabulary to manage the complexity of the topic and convey a style appropriate to the discipline and context as well as to the expertise of likely readers.</li> <li>e. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing.</li> <li>f. Provide a concluding statement or section that follows from and supports the information or explanation presented (e.g., articulating implications or the significance of the topic).</li> </ul> | <p>2. Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes.</p> <ul style="list-style-type: none"> <li>a. Introduce a topic and organize complex ideas, concepts, and information so that each new element builds on that which precedes it to create a unified whole; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension.</li> <li>b. Develop the topic thoroughly by selecting the most significant and relevant facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic.</li> <li>c. Use varied transitions and sentence structures to link the major sections of the text, create cohesion, and clarify the relationships among complex ideas and concepts.</li> <li>d. Use precise language, domain-specific vocabulary and techniques such as metaphor, simile, and analogy to manage the complexity of the topic; convey a knowledgeable stance in a style that responds to the discipline and context as well as to the expertise of likely readers.</li> <li>e. Provide a concluding statement or section that follows from and supports the information or explanation provided (e.g., articulating implications or the significance of the topic).</li> </ul> |
| <p>3. (See note; not applicable as a separate requirement)</p>   | <p>3. (See note; not applicable as a separate requirement)</p>  | <p>3. (See note; not applicable as a separate requirement)</p>   |

**Note:** Students' narrative skills continue to grow in these grades. The Standards require that students be able to incorporate narrative elements effectively into arguments and informative/explanatory texts. In history/social studies, students must be able to incorporate narrative accounts into their analyses of individuals or events of historical import. In science and technical subjects, students must be able to write precise enough descriptions of the step-by-step procedures they use in their investigations or technical work that others can replicate them and (possibly) reach the same results.



## Writing Standards for Literacy in History/Social Studies, Science, and Technical Subjects 6–12

WHST

| Grades 6–8 students:   | Grades 9–10 students:  | Grades 11–12 students:   |
|--|--|--|
| <b>Production and Distribution of Writing</b>  |  |  |
| 4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.  | 4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.  | 4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.  |
| 5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed.  | 5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience.  | 5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience.  |
| 6. Use technology, including the Internet, to produce and publish writing and present the relationships between information and ideas clearly and efficiently.   | 6. Use technology, including the Internet, to produce, publish, and update individual or shared writing products, taking advantage of technology's capacity to link to other information and to display information flexibly and dynamically.  | 6. Use technology, including the Internet, to produce, publish, and update individual or shared writing products in response to ongoing feedback, including new arguments or information.  |
| <b>Research to Build and Present Knowledge</b>   |  |  |
| 7. Conduct short research projects to answer a question (including a self-generated question), drawing on several sources and generating additional related, focused questions that allow for multiple avenues of exploration.   | 7. Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.  | 7. Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.  |
| 8. Gather relevant information from multiple print and digital sources, using search terms effectively; assess the credibility and accuracy of each source; and quote or paraphrase the data and conclusions of others while avoiding plagiarism and following a standard format for citation. | 8. Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the usefulness of each source in answering the research question; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and following a standard format for citation. | 8. Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the strengths and limitations of each source in terms of the specific task, purpose, and audience; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and overreliance on any one source and following a standard format for citation. |
| 9. Draw evidence from informational texts to support analysis, reflection, and research.   | 9. Draw evidence from informational texts to support analysis, reflection, and research.   | 9. Draw evidence from informational texts to support analysis, reflection, and research.   |
| <b>Range of Writing</b>  |  |  |
| 10. Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.   | 10. Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.   | 10. Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.   |

## Appendix B: Standards for Mathematical Practice

### Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

#### **1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

(NGA Center and CCSSO 2016b, 6 – 8)



**2 Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

**3 Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions,

(NGA Center and CCSSO 2010b, 6–8)

communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

#### **4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

**5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

**6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

(NGA Center and CCSSO 2010b, 6–8)



**7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

**8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

**Connecting the Standards for Mathematical Practice to the Standards for**

**Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content**

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

(NGA Center and CCSSO 2010b, 6–8)